Extreme Parties and Political Rents

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March 2011

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Abstract

We study the rent-seeking behaviour of political parties in a proportional representation system, where the final policy choice of the parliament is a weighted average of parties' policy positions, weights being their vote shares. We find that parties' policy preferences and their rent levels are strongly linked. Our main result is that an extreme party chooses a higher rent level than a moderate party, except in some cases of unlikely distributions of parties. An extreme party has more policy influence than a moderate party since it pulls the final policy towards its position more than a moderate party. Hence, a voter is ready to pay more rents to an extreme party in exchange of a greater policy influence. Furthermore, note that the voter does not need to be an extremist to vote for an extreme party. She is acting strategically in order to influence the final policy in her advantage as much as possible. In turn, this strategic behaviour of voters allow more extreme parties to earn higher rent levels.

Keywords: electoral competition, rent-seeking political parties, proportional representation system

JEL Classication: D72, D73, D78

1 Introduction

An important aspect of political power is that it gives access to public funds. Hence, abuse of political power is a potential problem. Clearly, voters would like parties to be honest, but they also have preferences on the policy choice of the parliament. The questions we ask are: Does political competition eliminate rents? And if not, which are the parties that will misbehave?

Our model consists of office-motivated parties who seek as large rents as possible in the context of a proportional representation system where each political party gets a number of seats proportional to its vote share. To put it differently, we do not assume exogenously that some parties are honest while some are not. Instead, the parties' rent levels are the result of the equilibrium. We find an important relationship between parties' policy preferences and their rent levels. More clearly, our most interesting result is that the more extreme a party is, the higher its rent level, except in some cases of unlikely distributions of parties. The reason is that an extreme party has more policy influence than a moderate party since it pulls the final policy towards its position more than a moderate party. This is due to our assumption, which is discussed below, that the final policy choice of the parliament is a weighted average of parties' policy positions. Hence, a voter is ready to vote for an extreme party with higher rent levels, since this way the voter has more policy influence and gets a closer final policy to her position, compared to the case where she votes for a more moderate party. In other words, a voter is ready to pay more rents to an extreme party in exchange of a higher policy influence. Furthermore, note that a voter may vote for an extreme party even if she is not extremist herself. She is acting strategically in order to influence the final policy to her advantage as much as possible. Clearly, that is exactly this strategic behavior of voters which permits more extreme parties to earn higher rent levels.

The existence of this type of strategic voting is confirmed empirically by Kedar (2005). This paper distinguishes two distinct drivers included in a voter's decision: proximity voting and compensational voting. The first one implies simply that a voter prefers voting for the party closest to her preferences. Compensational voting, on the other hand, means that a voter takes into account the effect of her vote on the final policy outcome and so, as in our setup, she prefers voting for more extreme parties than her own ideology in order to affect the policy outcome as much as possible in her advantage. The paper measures the impact of each motive using data from Great Britain and Canada which have majoritarian elections, and, Norway and the Netherlands which have proportional representation systems. Whereas, in the first two countries, there are almost always single-party governments which need little compromise with other parties, Norway and the Netherlands are more consensual democracies where the final policy outcome is a result of compromises between parties due to the lack of a single-party government. Hence, we would expect a greater impact of compensational voting for the last two countries, and this is exactly what is found in the data based on electoral surveys and election results.

In this paper, we assume that the final policy choice of the parliament is a weighted average of parties' policy positions, weights being their vote shares. This amounts to say that even parties out of the single-party or coalition government have influence on the final policy choice of the parliament. Contrary to the plurality rule with the winner-takes-all rule, the proportional representation system is meant to reflect better the different policy views of the population and to let these different views have a say on the important policy choices. For instance, Anderson and Guillory (1997) show that voters of opposition parties are satisfied with democracy in consensual democracies more than in majoritarian ones. Their analysis makes use of Lijphart (1984, 1994)'s consensus-majority index of democracies of which a very important criteria is the type of electoral system: Proportional representation system leads to a higher consensus score than plurality rule. Other criteria are the proportion of minimal winning cabinets, executive dominance, effective number of parties and the number of issue dimensions. The reason of the higher satistaction of opposition voters in consensual democracies is probably that opposition has more influence on policy-making in consensual democracies. Indeed, Powell (1989) finds high correlation in the expected direction between "effective representation in policy-making", which is defined as the percentage of voters represented by a political party influencing the policy-making process, and the type of legislative committee rules (consensual vs. majoritarian) and of electoral rules (proportional representation vs. plurality rule)[1]. However, most theoretical models of the proportional representation system assume that the final policy is only decided by the (possibly coalition) government[2]. Other papers using the same assumption as ours include Ortuno-Ortin (1997) and De Sinopoli and Iannantuoni (2007). De Sinopoli and Iannantuoni (2007) note that this assumption would be a result of a utilitarian solution of a bargaining process among all parties or a result of coalition formation procedure when the status quo is quite negative for parties, as shown in Baron and Diermeier (2001). Ortuno-Ortin (1997) argues further that the standard

[1] Actually, in Powell (1989), the type of legislative rules and of electoral rules mostly overlap, i.e. countries with proportional representation system happen to have consensual legislative committee rules, and vice versa. The classification of committee rules is based on Strom (1984). For instance, a proportionate distribution of committee chairs among the parties in the legislature leads to a higher consensus score.

[2] Examples of such papers include Austen-Smith and Banks (1988), Baron (1993), and Baron and Diermeier (2001).

Downsian approach where the party getting more than half of the votes has all authority on the final policy can be also criticized as unrealistic, given that "it is hard to believe that it makes no difference to win with 51% or to win with 90% of the votes". Hence, he sees these two different approaches, i.e. the Downsian and his (and also ours), as "polar cases that should be understood before analyzing more realistic ones".

Besides predicting that in proportional representation systems, extreme parties extract greater rents than moderate parties, we predict that a proportional representation system will not be able to eliminate political rents. On the contrary, a plurality rule where the majority has a dominant position in terms of policy decision making would be able to eliminate political rents unless there is uncertainty about voters' preferences as shown in Polo (1998). Hence, we predict higher political rents in proportional representation systems, which is confirmed empirically by Kunicova and Rose-Ackerman (2005) and Persson, Tabellini and Trebbi (2003). However, they propose different explanations from ours. The dependent variable of these two papers is corruption. As they argue, although corruption is more comprehensive than political rents, it is a relevant proxy. Indeed, Kunicova and Rose-Ackerman (2005) distinguish between two types of corrupt rent-seeking, namely by elected officials and by appointed bureaucrats. They define the first one as "the misuse of public office for private financial gain by an elected official". What we analyze in this paper fits well in this definition. Myerson (1993) studies the same problem as ours and calls it corruption. His paper focuses on coordination problems of voters that may result in an equilibrium with positive rents for political parties. Barro (1973) and Ferejohn (1986) analyze the case of holding an incumbent party accountable by way of threating her by not being re-elected in case of her excessive power abuse.

The closest paper to ours is De Sinopoli and Iannantuoni (2007) which also studies a proportional representation system where the final policy choice is the weighted average of political parties' ideal policies, weights being their vote shares. However, they do not consider political rent extraction problem. The originality of our paper is that we introduce a crucial attribute to parties other than their policy preferences, namely their rent levels. In their paper, in equilibrium, only two most extreme parties get votes since they have more policy influence than moderate parties. By introducing rents into the story, in equilibrium, we find that every party gets votes, which is empirically a more realistic finding that only two most extreme parties get votes. The reason is that, in our model, although more moderate parties have less policy influence than more extreme parties, they can still attract votes by seeking lower rent levels than the extreme ones. Moreover, this intuition explains also our finding that extreme parties get higher rent levels. As made precise above, another paper with the same assumption on the final policy is Ortuno-Ortin (1997). His focus is the strategical policy choice of two ideological political parties as a way to influence the final policy. This paper does not consider either political rent extraction. Moreover, it is assumed that voters vote sincerely.

Our model is as simple as possible. There are a number of office-motivated parties with different policy preferences on a unidimensional policy parameter. First, each party chooses a rent level. Second, there are elections. Finally, the policy choice is the weighted average of the ideal positions of political parties, weights being their vote shares. In addition, each party in the parliament receives rents equal to its chosen level weighted by its vote share. Hence, its chosen rent level is not the same as its payoff. Given its chosen rent level, a party gets higher rent with higher vote shares. Its chosen rent level can be interpreted as its eagerness to extract rents. Then, its final payoff depends also on its political power which we measure by its vote share. Before her decision to vote, a voter knows the policy preferences and the chosen rent levels of all political parties[3]. In equilibrium, her decision involves a trade-off: while voting for extreme parties pulls the final policy more towards her ideal policy, voting for moderate parties costs less in terms of rents, since moderate parties choose lower rent levels, as explained above.

There is a similarity between our model and the Hotelling model of horizantally differentiated goods. Political parties are like firms whose localization (ideal policy in our framework) is given, who set prices and attract demands. However, there is a crucial difference between consumers in the Hotelling model and voters in our model: Whereas consumers' preferences depend only on the localization and on the price of the firm from which they buy, voters' preferences depend on the weighted average of all parties' localizations (ideal policies) and prices (rents), weights being parties' demands (vote shares).

We describe the model in section 2. In section 3, we analyze the equilibrium. In the original model, we assume that voters are uniformly distributed. Section 4 discusses the case of a non-uniform distribution of voters. Section 5 concludes. Most proofs are relegated to the appendix.

2 The Model

There are m political parties and a continuum of voters. There is a unidimensional policy choice variable $p \in [-1, 1]$. Parties are indexed according to their ideal policy points such

[3]It is clearly not realistic to assume that a voter knows perfectly parties' rent levels. However, as Myerson (1993) argues, it is the most restrictive setup in terms of parties' ability to extract rents, so if they are able to extract rents even in this situation, they are also able a fortiori in more realistic setups.

that $p_i < p_{i+1}$ for every $i = 1, 2, ..., m-1$, where p_i is the ideal policy point of party i. We assume that no two parties have identical ideal policy points. The ideal policy points of voters of mass 1 are uniformly distributed on the segment $[-1, 1]$.

The timing of the game is as follows:

First, each party i selects and announces $r_i \geq 0$, where r_i represents party i's rent level.

Second, each voter votes for one of the political parties.

Finally, the final policy is set to be $p = \sum_{i=1}^{m} \alpha_i p_i$, where α_i is party i's vote share. In other words, the final policy is the weighted average of parties' ideal policy points, weights being the vote shares. Each party i gets $\alpha_i r_i$ as rent.

The payoff of a political party i is $\alpha_i r_i$.

The utility function of a voter is $-(p-x)^2 - r$,where x is the ideal policy point of the voter, and $r = \sum_{i=1}^{m} \alpha_i r_i$ is the total rent of the political parties.

Our solution concept is subgame perfect Nash equilibrium subject to an equilibrium refinement for the voting stage. Hence, we proceed by backwards induction.

3 Equilibrium

3.1 The Voting Stage

Since there is a continuum of voters, a deviation of a lone voter does not change the outcome. Hence, we need an equilibrium refinement for the voting stage. We assume that a voter considers herself as being of an arbitrarily small mass $\varepsilon > 0$ so that she can change the election outcome. In equilibrium, there should be no voter who would like to deviate for any $\varepsilon > 0$.

Define $e_{i,j}$ as

$$e_{i,j} = p + \frac{r_j - r_i}{2(p_j - p_i)}$$

Proposition 1 *Define $\underline{e}_i$ as $\underline{e}_i = \max_{j<i}\{e_{i,j}, -1\}$ for $i \neq 1$ and as $\underline{e}_i = -1$ for $i = 1$, and define $\overline{e}_i$ as $\overline{e}_i = \min_{j>i}\{e_{i,j}, 1\}$ for $i \neq m$ and as $\overline{e}_i = 1$ for $i = m$. Then, in any equilibrium and for any i, party i's vote share is*

$$\alpha_i = \max\{0, \frac{\overline{e}_i - \underline{e}_i}{2}\}$$

It is shown in the proof that the ideal policy of the indifferent voter between parties i and j is given by

$$e_{i,j} = p + \frac{r_j - r_i}{2\left(p_j - p_i\right)}$$

If $j > i$, i.e. if party j is to the right of party i, voters to the left of this boundary voter prefer party i to party j, and those to the right, party j to party i. The inverse is true for $j < i$. Hence, the result follows, given that a voter votes for party i if and only if she prefers it to any other party.

This result holds for any value of rents chosen at the previous stage of the game. Hence, it is possible that a party does not receive any vote. However, next, we look for an equilibrium of the rent decision stage and show that equilibrium rent levels are such that every party receives votes in equilibrium.

Note also that the vote share of party i is implicitly characterized by this proposition, since the final policy depends in its turn on this vote share.

3.2 The Rent Decision Stage

We start by proving that every party chooses a positive rent level and has votes in any equilibrium.

Proposition 2 *In any equilibrium, every party has a positive vote share and chooses a positive rent level.*

The vote share of a party i is $\alpha_i = \frac{e_{i,i+1} - e_{i,i-1}}{2}$ for any $i \neq 1, m$. The vote shares of parties 1 and m are respectively $\alpha_1 = \frac{e_{1,2} - (-1)}{2}$ and $\alpha_m = \frac{1 - e_{m,m-1}}{2}$.

The intuition of this result is that although extreme parties have an advantage in terms of policy influence, moderate parties are also able to attract votes by proposing lower rent levels. Since every party receive votes, the general expression of party i's vote share in Proposition 1 reduces to the simple expression of Proposition 2.

Now, consider the expression determining the boundary voter between parties i and $i+1$:

$$e_{i,i+1} = p + \frac{r_{i+1} - r_i}{2\left(p_{i+1} - p_i\right)}$$

We see that if both parties i and $i+1$ announce the same rent level, i.e. if $r_i = r_{i+1}$, then the ideal point of the boundary voter between parties i and $i+1$ is p. To interpret this, assume that both p_i and p_{i+1} are to the left of p. Given that both parties announce the same rent level, every voter to the left of p will prefer to vote for party i rather than to vote for party $i+1$, since a vote for party i is pulling the final policy more to the left. Conversely, every voter to the right of p will prefer to vote for party $i+1$ rather than to vote for party i, since their vote is pulling less the final policy choice to the left.

We also see that parties can change the location of the indifferent voter and increase their vote shares by decreasing their rent levels. To illustrate, assume again that both p_i and p_{i+1} are to the left of p. In that case, party i has an advantage to attract votes of voters to the left of p, as explained above. Hence, party $i+1$ has to require less rents than party i in order to attract some of these votes. If this is the case, since they will pay less rents, some voters to the left of p votes for party $i+1$ although this party's policy influence is less important.

One other observation is that the sensitivity of the choice of voters between party i and party $i+1$ to the rent levels increases as the difference of the two parties' ideal policy points decreases. Closer ideal points means less important difference in policy influence, so voters are more sensitive to parties' rent levels.

The vote share of party i, $i = 2, ..., m-1$, is

$$\alpha_i = \frac{1}{2}\left(e_{i,i+1} - e_{i,i-1}\right) = \frac{1}{2}\left(\frac{r_{i+1} - r_i}{2\left(p_{i+1} - p_i\right)} - \frac{r_i - r_{i-1}}{2\left(p_i - p_{i-1}\right)}\right) \tag{1}$$

Clearly, as r_i increases or as r_{i-1} or r_{i+1} decreases, party i's vote share decreases.

We remark that the vote share of party i, for $i = 2, ..., m-1$, does not depend on p. To understand why, assume that all p_{i-1}, p_i and p_{i+1} are to the left of p and focus on party i. Party i competes with party $i-1$ and party $i+1$. It has a policy advantage against party $i+1$ since she pulls the final policy more to the left. Conversely, it has a disadvantage against party $i-1$. This advantage and disadvantage cancel out each other in the case of uniform distribution of voters and consequently, the vote share of party i does not depend on p.

The vote shares of parties 1 and m are respectively

$$\alpha_1 = \frac{1}{2}\left(e_{1,2} - (-1)\right) = \frac{1}{2}\left(p + \frac{r_2 - r_1}{2\left(p_2 - p_1\right)} + 1\right) \tag{2}$$

$$\alpha_m = \frac{1}{2}\left(1 - e_{m,m-1}\right) = \frac{1}{2}\left(1 - \left(p + \frac{r_m - r_{m-1}}{2\left(p_m - p_{m-1}\right)}\right)\right) \tag{3}$$

We remark that party 1's vote share is increasing and party m's vote share is decreasing with p. The reason is that when the final policy moves to the right, the policy influence of party 1 (respectively party m) becomes more (respectively less) important.

Now, we have a result on the final policy.

Proposition 3 *In any equilibrium, the final policy is given by*

$$p = \frac{1}{2 + p_m - p_1}\left(p_1 + p_m + \frac{r_1 - r_m}{2}\right) \tag{4}$$

We see that the final policy does not depend directly on the ideal policy points nor on the rent levels of parties $2, 3, ..., m-1$, but only indirectly through r_1 and r_m. Disregarding the rent levels, every voter would vote either for party 1 or for party m given that their policy influence are greater than other parties. Other parties get votes since they propose lower rent levels than the most influential two parties. Finally, the direct effect of this competition between these parties on the final policy choice cancel out.

To illustrate this result, consider that r_i is marginally increased for some party $i = 2, 3, ..., m-1$. Then, the vote share of party i decreases by $\frac{1}{2}\left(\frac{1}{2(p_i - p_{i-1})} + \frac{1}{2(p_{i+1} - p_i)}\right)$, the vote shares of party $i-1$ and of party $i+1$ increase respectively by $\frac{1}{2}\left(\frac{1}{2(p_i - p_{i-1})}\right)$ and $\frac{1}{2}\left(\frac{1}{2(p_{i+1} - p_i)}\right)$. The final policy change would be in this case $-\frac{1}{2}\left(\frac{1}{2(p_i - p_{i-1})} + \frac{1}{2(p_{i+1} - p_i)}\right)p_i + \frac{1}{2}\left(\frac{1}{2(p_i - p_{i-1})}\right)p_{i-1} + \frac{1}{2}\left(\frac{1}{2(p_{i+1} - p_i)}\right)p_{i+1} = 0$, meaning that the final policy does not change.

Now, consider the maximization problem of party i:

$$\max_{r_i} \alpha_i r_i$$

For $i = 2, 3, ..., m-1$, after replacing α_i, this becomes

$$\max_{r_i} \frac{1}{2}\left(\frac{r_{i+1} - r_i}{2(p_{i+1} - p_i)} - \frac{r_i - r_{i-1}}{2(p_i - p_{i-1})}\right) r_i$$

which gives the following reaction function

$$r_i = \frac{1}{2}\left(\frac{(p_i - p_{i-1})\, r_{i+1} + (p_{i+1} - p_i)\, r_{i-1}}{p_{i+1} - p_{i-1}}\right) \tag{5}$$

Party i's rent level is increasing with its two adjacant parties' rent levels. When an adjacent party chooses a higher rent level, party i's vote share increases and therefore it becomes more eager to choose a higher rent level.

Moreover, we see that the rent level of party i is a half weighted average of the rent levels of its two adjacent parties. The weight is bigger for the farther adjacent party, since the competition with it is less harsh, given that it is not as good as the closer adjacent party as a substitute to party i in terms of policy influence.

With this choice of the rent level, the vote share of party i becomes

$$\alpha_i = \frac{1}{2}\left(\frac{r_{i+1}}{4\left(p_{i+1}-p_i\right)} + \frac{r_{i-1}}{4\left(p_i - p_{i-1}\right)}\right) \tag{6}$$

We see that party i's vote share increases when its adjacent parties' ideal policy points get closer to its. In this case, the sensitivity of the vote share to the rent level increases, then party i decreases its rent level sizably, and its vote share increases.

The problem of party 1 is

$$\max_{r_1} \alpha_1 r_1$$

Replacing the expression for p into equation (2), this maximization yields

$$r_1 = \frac{2+p_m-p_1}{2\left(2+p_m-p_2\right)} r_2 - \frac{\left(p_2-p_1\right)}{2\left(2+p_m-p_2\right)} r_m + \frac{\left(p_2-p_1\right)\left(2+2p_m\right)}{2+p_m-p_2} \tag{7}$$

We see that the rent levels of parties 1 and 2 are strategic complements as for any other pair of adjacent parties. However, when party m chooses a lower rent level, the final policy moves to the right, which gives a higher advantage to party 1 and so enables it to choose a higher rent level. For party 1, there are two effects of an increase of its own rent level on its vote share. The first and dominant one is true for every party: a higher rent level induces some voters to vote for the adjacent party. The second effect is more subtle: when r_1 increases, the final policy moves to the right, which gives a higher policy influence and therefore a greater advantage to party 1.

With this choice of the rent level, the vote share of party 1 becomes

$$\alpha_1 = \frac{1}{2}\left(\frac{r_2}{4\left(p_2-p_1\right)} - \frac{r_m}{4\left(2+p_m-p_1\right)} + \frac{1+p_m}{2+p_m-p_1}\right)$$

Similarly as above, the first term tells that party 1's vote share increasing with its adjacent party's rent level. The intuition of the second term is more subtle. When r_m increases, the final policy moves to the left. Then party 1's policy influence is smaller and therefore its vote share decreases. Another remark about this term is that as the distance between p_1 and p_m increases, the effect of r_m on the final policy decreases, as can be seen from equation (4). Hence, for a given level of r_m, party 1 has less policy advantage and becomes more careful when choosing a rent level and consequently its vote share increases.

The problem of party m is

$$\max_{r_m} \alpha_m r_m$$

Replacing the expression for p into equation (3), this maximization yields

$$r_m = \frac{2 + p_m - p_1}{2(2 + p_{m-1} - p_1)} r_{m-1} - \frac{(p_m - p_{m-1})}{2(2 + p_{m-1} - p_1)} r_1 + \frac{(p_m - p_{m-1})(2 - 2p_1)}{2 + p_{m-1} - p_1} \tag{8}$$

With this choice of the rent level, the vote share of party m becomes

$$\alpha_m = \frac{1}{2}\left(\frac{r_{m-1}}{4(p_m - p_{m-1})} - \frac{r_1}{4(2 + p_m - p_1)} + \frac{1 - p_1}{2 + p_m - p_1}\right)$$

The remarks for party m are the symmetric ones of those for party 1.

To find the equilibrium, we need to solve m equations with m unknowns. However, solving this analytically turns out to be very complicated and not so fruitful in terms of intuitions even for small numbers of parties. Hence, instead, we will give some numerical examples in the next subsection with the help of a mathematics software package.

Before going to numerical examples, first, we prove the existence and uniqueness of the equilibrium[4]. Second, we show analytically an important result in case of a symmetric distribution of parties' ideal policy points: extreme parties choose higher rent levels and extract more rents. We should emphasize that symmetric distribution of parties is not a necessary but a sufficient condition for this result. As it will be seen and discussed with numerical examples, extreme parties choose higher rent levels except for unlikely distributions of parties.

Proposition 4 *At the rent decision stage, an equilibrium exists and it is unique.*

Proposition 5 *When parties' ideal policy points are symmetrically distributed around 0, (i) a party chooses a rent level at least twice as high as its more moderate adjacent party. (ii) a party's vote share is higher than its more moderate adjacent party for $m \geq 4$. (iii) a party's payoff is at least twice as high as its more moderate adjacent party for $m \geq 4$.*

We note that parts (ii) and (iii) of Proposition 5 do not hold for $m = 3$. However, part (i) holds, i.e. the extreme party chooses a rent level at least twice as high as the moderate party in the case of 3 symmetric parties. In addition, it can be easily shown that the extreme party's vote share is lower than the moderate party's vote share. However, the payoff of the extreme party is higher than the payoff of the moderate party, but not twice as high. The first table of numerical examples presents an illustration.

[4]The proofs of the existence and of the uniqueness of equilibrium are similar to the ones in Neven (1987), a paper on product differentiation.

3.2.1 Numerical Examples

Below, we give some numerical examples for 3-party case.

$m = 3$	$p_1 = -1$	$p_2 = 0$	$p_3 = 1$
rent levels	1.6	0.8	1.6
vote shares	30%	40%	30%
payoffs	0.48	0.32	0.48
final policy		0	

Table 1: 3-party case with $p_1 = -1$, $p_2 = 0$, $p_3 = 1$ and uniform distribution of voters

$m = 3$	$p_1 = -1$	$p_2 = 0.2$	$p_3 = 1$
rent levels	2.00	0.77	1.23
vote shares	29.2%	40.0%	30.8%
payoffs	0.58	0.31	0.38
final policy		0.10	

Table 2: 3-party case with $p_1 = -1$, $p_2 = 0.2$, $p_3 = 1$ and uniform distribution of voters

$m = 3$	$p_1 = -1$	$p_2 = 0.9$	$p_3 = 1$
rent levels	3.71	0.16	0.14
vote shares	25.6%	41.3%	33.0%
payoffs	0.95	0.06	0.04
final policy		0.45	

Table 3: 3-party case with $p_1 = -1$, $p_2 = 0.9$, $p_3 = 1$ and uniform distribution of voters

Table 1 presents a case of a symmetric policy configuration. In accordance with part (i) of Proposition 5, the rent level chosen by the two extreme parties are twice as large as the one chosen by the moderate party. Table 2 shows that as the ideal policy of the moderate party approaches the ideal policy of the right-wing extreme party, the rent levels chosen by both of these two parties decrease. This is because they are now better substitutes than in the first case. Hence, the competition between them gets harsher and results in lower chosen rent levels. Moreover, since the vote share becomes more sensitive to the rent level for the right-wing extreme party, this party decreases its rent level in such a way that its vote share increases with respect to the first case. On the other hand, the rent level of the left-wing extreme party increases since the moderate party becomes a worse substitute for it. Finally, we see that the final policy is not the median policy anymore.

Table 3 presents an extreme case where the middle party is almost as extreme as the right-wing extreme party. Clearly, the rents of these two parties decrease dramatically compared to other cases, due to high substituability and therefore strong competition between them. In other words, having a good substitute decreases significantly the ability to extract rents. Moreover, the right-wing extreme party chooses a lower rent level and gets a lower payoff than the slightly more moderate party. This exceptional situation arises because the final policy is quite to the right ($p = 0.45$) and voters to the left of it prefer the moderate party to the right-wing extreme party for less negative policy influence. The reason why many of them do not vote simply for the left-wing party is that this party proposes a too high rent level as a result of her privileged situation, namely being extreme and having very bad substitutes. Fixing $p_1 = -1$ and $p_3 = 1$, this exceptional case where party 2 chooses a higher rent level than that of party 1 or of party 3 does not occur when $-0.71 \leq p_2 \leq 0.71$. Moreover, if we imagine a stage at the beginning of our game where parties choose their ideal policies, it is clear that the middle party would not choose this extreme policy and the "unreasonable" case of table 3 would not occur. Indeed, given the positions of the two extreme parties, the middle party would choose the median policy. In other words, in the extended version of the game, only the symmetric case represented by table 1 would be an equilibrium.

Below, we give some numerical examples for 5-party case.

$m = 5$	$p_1 = -1$	$p_2 = -0.5$	$p_3 = 0$	$p_4 = 0.5$	$p_5 = 1$
rent levels	0.63	0.18	0.09	0.18	0.63
vote shares	27.5%	18.0%	9.0%	18.0%	27.5%
payoffs	0.17	0.03	0.01	0.03	0.17
final policy	0				

Table 4: 5-party case with $p_1 = -1$, $p_2 = -0.5$, $p_3 = 0$, $p_4 = 0.5$, $p_5 = 1$ and uniform distribution of voters

$m=5$	$p_1=-1$	$p_2=-0.9$	$p_3=0$	$p_4=0.9$	$p_5=1$
rent levels	0.13	0.06	0.03	0.06	0.13
vote shares	32.2%	16.9%	1.7%	16.9%	32.2%
payoffs	0.04	0.01	0.00	0.01	0.04
final policy	0				

Table 5: 5-party case with $p_1 = -1$, $p_2 = -0.9$, $p_3 = 0$, $p_4 = 0.9$, $p_5 = 1$ and uniform distribution of voters

$m=5$	$p_1=-1$	$p_2=-0.42$	$p_3=0$	$p_4=0.42$	$p_5=1$
rent levels	0.72	0.18	0.09	0.18	0.72
vote shares	26.6%	18.2%	10.5%	18.2%	26.6%
payoffs	0.19	0.03	0.01	0.03	0.19
final policy	0				

Table 6: 5-party case with $p_1 = -1$, $p_2 = -0.42$, $p_3 = 0$, $p_4 = 0.42$, $p_5 = 1$ and uniform distribution of voters

Table 5 shows a situation where parties 2 and 4 are more extreme compared to those of table 4. Similar to the case of 3 parties, we see that the rent levels of extreme parties decrease drastically due to high substituability, nevertheless, they stay higher than those of more moderate parties. Moreover, for the same reason as in the case of 3 parties, their vote shares increase. We also note that the middle party is significantly hurt too. Since the rent levels of more extreme parties decrease significantly, the middle party has to decrease its rent level in order to be able to attract voters. Finally, if we imagine a stage at the beginning of our game where parties choose their ideal policies, the equilibrium would be represented by table 6.

4 Non-Uniform Distribution of Voters

In this section, we do not assume a uniform distribution of voters' ideal policy points in order to see the robustness of our results. We assume that voters' ideal policies are distributed on $[-1, 1]$ according to the cdf $F(\cdot)$ with the pdf $f(\cdot)$. We focus on the arguably more realistic case that the density of moderate voters is higher than that of extreme voters.

First of all, it is important to note that Propositions 1 and 2 hold for any distribution of voters' ideal policies, and not necessarily for a uniform distribution, except that the vote shares should be rewritten for a general distribution[5]. In other words, we know that in any equilibrium, every party has a positive vote share and chooses a positive rent level, and that the ideal points of the boundary voters do not depend on the type of the distribution.

Next, we present an important result which holds for any distribution of voters.

Proposition 6 *When parties' ideal policy points are symmetrically distributed around 0, a party chooses a higher rent level than its more moderate adjacent party for any distribution of voters' ideal policies in any equilibrium.*

Now, we focus on the maximization problem of party i, $i \neq 1, m$, in order to see the implications of a non-uniform distribution of voters[6]. This problem is

$$\max_{r_i}(F(e_{i,i+1}) - F(e_{i,i-1}))r_i$$

with the following first-order condition

$$(F(e_{i,i+1}) - F(e_{i,i-1})) + r_i \left(f(e_{i,i+1})\frac{de_{i,i+1}}{dr_i} - f(e_{i,i-1})\frac{de_{i,i-1}}{dr_i} \right) = 0$$

Given that $\frac{de_{i,i+1}}{dr_i} = \frac{dp}{dr_i} - \frac{1}{2(p_{i+1}-p_i)}$ and $\frac{de_{i,i-1}}{dr_i} = \frac{dp}{dr_i} + \frac{1}{2(p_i-p_{i-1})}$, this implies

$$r_i = \frac{F(e_{i,i+1}) - F(e_{i,i-1})}{\left[\frac{1}{2(p_{i+1}-p_i)} f(e_{i,i+1}) + \frac{1}{2(p_i-p_{i-1})} f(e_{i,i-1})\right] - \left[\frac{dp}{dr_i}\left(f(e_{i,i+1}) - f(e_{i,i-1})\right)\right]}$$

The term in the numerator implies that with a denser distribution on moderate voters, moderate parties are more willing to choose high rent levels compared to the uniform distribution case, since they have higher vote shares and an increase in the rent level corresponds to a higher increase in their payoffs. The terms in the denominator measure the sensitivity of the vote share to the rent level. The first term shows an opposite effect to the previous one: Moderate parties are less willing to choose higher rent levels compared to the uniform distribution case because of a higher density of their voters. However, there is also the more involved second term.

[5]The expression $\alpha_i = \max\{0, \frac{\overline{e}_i - \underline{e}_i}{2}\}$ in Proposition 1 should be changed to $\alpha_i = \max\{0, F(\overline{e}_i) - F(\underline{e}_i)\}$. The expressions $\alpha_i = \frac{e_{i,i+1}-e_{i,i-1}}{2}$ for any $i \neq 1, m$, $\alpha_1 = \frac{e_{1,2}-(-1)}{2}$ and $\alpha_m = \frac{1-e_{m,m-1}}{2}$ in Proposition 2 should be changed respectively to $\alpha_i = F(e_{i,i+1}) - F(e_{i,i-1})$ for any $i \neq 1, m$, $\alpha_1 = F(e_{1,2})$ and $\alpha_m = 1 - F(e_{m,m-1})$.

[6]The problems of parties 1 and m are similar and will be skipped.

Before analyzing this term, we note that the final policy p is given by

$$p = \sum_{j=1}^{m} (F(e_{j,j+1}) - F(e_{j,j-1}))p_j$$

By the implicit function theorem, and after some manipulations, for $i \neq 1, m$,

$$\frac{dp}{dr_i} = \frac{f(e_{i,i+1}) - f(e_{i,i-1})}{2\left[1 + \sum_{j=1}^{m-1} f(e_{j,j+1})(p_{j+1} - p_j)\right]} \tag{9}$$

For the uniform distribution f, $\frac{dp}{dr_i} = 0$ for $i \neq 1, m$, since $f(e_{i,i+1}) = f(e_{i,i-1})$. Indeed, we have already discussed that the effect of the rent level of a party $i \neq 1, m$ on the final policy is nil in case of the uniform distribution. However, the subtle effect coming through the change in final policy already exists for parties 1 and m in the case of the uniform distribution. Now, this effect exists possibly for every party. For instance, if we assume that f is a concave distribution and $f(e_{i,i+1}) > f(e_{i,i-1})$ for party i, $i \neq 1, m$, then according to (9), an increase in r_i shifts the final policy p to the right. This means that $e_{i,i+1}$ moves less to the left and $e_{i,i-1}$ moves more to the right compared to the uniform distribution case. Since $f(e_{i,i+1}) > f(e_{i,i-1})$, the second term in the denominator implies a lower sensitivity of the vote share, which makes parties more willing to choose high rent levels. In other words, the effect represented the second term offsets partly the effect represented by the first term. Naturally, the first term is dominant so that a party loses votes when it increases its rent level[7].

All in all, the comparison between the uniform and non-uniform distribution cases does not give unambiguous results due to the existence of opposite effects. Next, we give two numerical examples for 3-party case where we assume a symmetric triangular distribution of voters on $[-1, 1]$. Hence, compared to the uniform distribution case, there are more moderate voters and less extreme voters.

Table 7 below presents a case of a symmetric policy configuration. As we already knew from Proposition 6 which holds for any distribution of voters, the rent level chosen by the two extreme parties are larger than that of the moderate party. Comparing this table with table 1 presenting the uniform distribution case, we see that the moderate party chooses a lower

[7]Replacing $\frac{dp}{dr_i}$ by its above value in the expressions of $\frac{de_{i,i+1}}{dr_i}$ and $\frac{de_{i,i-1}}{dr_i}$, it can be easily shown that $\frac{de_{i,i+1}}{dr_i} < 0$ and $\frac{de_{i,i-1}}{dr_i} > 0$.

$m = 3$	$p_1 = -1$	$p_2 = 0$	$p_3 = 1$
rent levels	1.07	0.58	1.07
vote shares	28.2%	43.4%	28.2%
payoffs	0.30	0.25	0.30
final policy	0		

Table 7: 3-party case with $p_1 = -1$, $p_2 = 0$, $p_3 = 1$ and symmetric triangular distribution of voters around 0

$m = 3$	$p_1 = -1$	$p_2 = 0.2$	$p_3 = 1$
rent levels	1.39	0.56	0.80
vote shares	27.8%	43.6%	28.5%
payoffs	0.39	0.24	0.23
final policy	0.09		

Table 8: 3-party case with $p_1 = -1$, $p_2 = 0.2$, $p_3 = 1$ and symmetric triangular distribution of voters around 0

rent level. Hence, it seems that the dominant effect is the increased sensitivity of vote share due to higher density of moderate voters. This is reflected on the extreme parties which also choose lower rent levels. It can be also remarked that the payoff difference between moderate and extreme parties is smaller thanks to higher density of moderate voters. Table 8 presents the same policy configuration as table 2 of the uniform distribution case. Similarly, the rent levels of parties 2 and 3 decrease and that of party 1 increases. Similarly again, the vote share of party 3 increases and that of party 1 decreases. More importantly, although the payoff of party 2 is higher than the payoff of party 3, it is still true that extreme parties choose higher rent levels.

5 Conclusion

Given that every party in parliament has a say on the final policy choice, which is a plausible assumption in the context of proportional representation systems meant to reflect different views in the society, we show that extreme parties are more eager to extract rents than moderate parties, except in some cases of unlikely distributions of parties. This is due to the fact that extreme parties have more policy influence than moderate parties, since they are able to pull the final policy towards their ideal policy choices to a greater extent. Hence, a voter is ready to pay more rents to extreme parties in return of higher policy influence.

Moreover, since moderate parties are also able to attract votes by proposing lower rent levels than extreme parties, every party has positive vote share in equilibrium. With the same assumption on the determination of the final policy, De Sinopoli and Iannantuoni (2007) arrives to the result that only two most extreme parties obtain votes, since the authors do not consider political rents. Given that for a proportional representation system, every party getting votes is more likely than only two most extreme parties getting votes, we observe that taking into account political rents is also helpful to obtain more realistic results about the voting equilibrium.

6 Appendix

Proof of Proposition 1: Suppose we have a voting equilibrium where the profile of parties' vote shares is $(\alpha_1, ..., \alpha_i, ..., \alpha_m)$, the final policy is p and political parties' total rent is r. Consider that a voter x of party i deviates and votes for party $j \neq i$. Then the vote shares of parties i and j would change to

$$\alpha_i' = \alpha_i - \varepsilon$$

and

$$\alpha_j' = \alpha_j + \varepsilon$$

Then, the new final policy p' and the new total rent r' would be respectively

$$p' = p - \varepsilon p_i + \varepsilon p_j$$

and

$$r' = r - \varepsilon r_i + \varepsilon r_j$$

If the initial situation is an equilibrium, it should be that this deviation is not profitable for voter x. Hence,

$$-(p' - x)^2 - r' \leq -(p - x)^2 - r$$

Equivalently,

$$2x(p_j - p_i) \leq 2p(p_j - p_i) + (r_j - r_i) + \varepsilon(p_j - p_i)^2$$

Since this should hold for any $\varepsilon > 0$, it reduces to

$$2x(p_j - p_i) \leq 2p(p_j - p_i) + (r_j - r_i)$$

Given the definition of $e_{i,j}$, this is equivalent to

$$x \leq e_{i,j}$$

if $j > i$, and to

$$x \geq e_{i,j}$$

if $j < i$.

Since voter x should not deviate for any $j \neq i$, it should be that, for $i \neq 1, m$,

$$\max_{j<i}\{e_{i,j}, -1\} \leq x \leq \min_{j>i}\{e_{i,j}, 1\}$$

for $i = 1$,

$$-1 \leq x \leq \min_{j>i}\{e_{i,j}, 1\}$$

for $i = m$,

$$\max_{j<i}\{e_{i,j}, -1\} \leq x \leq 1$$

Q.E.D.

Proof of Proposition 2: Assume a party i has votes in equilibrium, then it can choose $r_i > 0$, since its vote share is continuous in r_i.

Call $V \subseteq \{1, ..., m\}$ the subset of parties having positive vote shares in equilibrium. Call parties in V as $\mathbf{1}, \mathbf{2}, ..., \mathbf{k}, ..., \mathbf{K}$ such that $p_{\mathbf{k}} < p_{\mathbf{k+1}}$.

If $\mathbf{k} \in V$, then $e_{\mathbf{k},\mathbf{k+1}} > e_{\mathbf{k},\mathbf{k-1}}$, since otherwise $\overline{e}_{\mathbf{k}} < \underline{e}_{\mathbf{k}}$. Then, $\alpha_{\mathbf{k}} = \frac{e_{\mathbf{k},\mathbf{k+1}} - e_{\mathbf{k},\mathbf{k-1}}}{2}$ for every $\mathbf{k} \in V$, $\mathbf{k} \neq \mathbf{1}, \mathbf{K}$, $\alpha_{\mathbf{1}} = \frac{e_{\mathbf{1},\mathbf{2}} - (-1)}{2}$ and $\alpha_{\mathbf{K}} = \frac{1 - e_{\mathbf{K},\mathbf{K-1}}}{2}$.

Assume a party i does not have votes (i.e. $i \notin V$) and $p_{\mathbf{k}} < p_i < p_{\mathbf{k+1}}$ for some $\mathbf{k}, \mathbf{k+1} \in V$. This party i could have votes if and only if $e_{i,\mathbf{k+1}} - e_{i,\mathbf{k}} > 0$. Equivalently,

$$\frac{r_{\mathbf{k+1}} - r_i}{2\left(p_{\mathbf{k+1}} - p_i\right)} - \frac{r_i - r_{\mathbf{k}}}{2\left(p_i - p_{\mathbf{k}}\right)} > 0$$

Equivalently,

$$r_i < \frac{\left(p_i - p_{\mathbf{k}}\right) r_{\mathbf{k+1}} + \left(p_{\mathbf{k+1}} - p_i\right) r_{\mathbf{k}}}{p_{\mathbf{k+1}} - p_{\mathbf{k}}}$$

Since $r_{\mathbf{k}} > 0$ and $r_{\mathbf{k+1}} > 0$, such $r_i > 0$ exists. Hence, party i is able to have votes and to choose a positive rent level.

Assume now that $i \notin V$ and $p_i < p_{\mathbf{1}}$. This party i could have votes if and only if

$e_{i,\mathbf{1}} - (-1) > 0$. Equivalently,

$$\frac{r_{\mathbf{1}} - r_i}{2\left(p_{\mathbf{1}} - p_i\right)} - (-1) > 0$$

Equivalently,

$$r_i < r_{\mathbf{1}} + 2\left(p_{\mathbf{1}} - p_i\right)$$

Since $r_{\mathbf{1}} > 0$ and $p_{\mathbf{1}} > p_i$, such $r_i > 0$ exists. Hence, party i is able to have votes and to choose a positive rent level.

Assume now that $i \notin V$ and $p_i > p_{\mathbf{K}}$. This party i could have votes if and only if $1 - e_{i,\mathbf{K}} > 0$. Equivalently,

$$1 - \frac{r_i - r_{\mathbf{K}}}{2\left(p_i - p_{\mathbf{K}}\right)} > 0$$

Equivalently,

$$r_i < r_{\mathbf{K}} + 2\left(p_i - p_{\mathbf{K}}\right)$$

Since $r_{\mathbf{K}} > 0$ and $p_i > p_{\mathbf{K}}$, such $r_i > 0$ exists. Hence, party i is able to have votes and to choose a positive rent level.

Hence, to sum up, in equilibrium, every party get votes and choose positive rent levels.

The vote share for a party i is $\alpha_i = \frac{e_{i,i+1} - e_{i,i-1}}{2}$ for any $i \neq 1, m$. The vote shares of parties 1 and m are respectively $\alpha_1 = \frac{e_{1,2} - (-1)}{2}$ and $\alpha_m = \frac{1 - e_{m,m-1}}{2}$. **Q.E.D.**

Proof of Proposition 3: The final policy is defined by $p = \sum_{i=1}^{m} \alpha_i p_i$. After replacing the vote shares,

$$p = \frac{1}{2}(e_1 + 1)p_1 + \sum_{k=2}^{m-1} \frac{1}{2}\left(\frac{r_{i+1} - r_i}{2\left(p_{i+1} - p_i\right)} - \frac{r_i - r_{i-1}}{2\left(p_i - p_{i-1}\right)}\right) p_i + \frac{1}{2}(1 - e_{m-1})p_m$$

We get the result after simplifications. **Q.E.D.**

Proof of Proposition 4: To prove the existence of an equilibrium, it is sufficient to show that the vote share function of a party i, i.e. α_i, is concave in r_i for every i. Because, if this holds, then the objective function of a party i, $\alpha_i r_i$ is also concave in r_i. Then a theorem by Rosen (1965) guarantees the existence of an equilibrium.

For $i = 2, 3, ..., m-1$,

$$\alpha_i = \frac{1}{2}\left(\frac{r_{i+1} - r_i}{2\left(p_{i+1} - p_i\right)} - \frac{r_i - r_{i-1}}{2\left(p_i - p_{i-1}\right)}\right)$$

given that adjacent parties have also some votes. In this case, α_i is linear in r_i, and

$$\frac{\partial \alpha_i}{\partial r_i} = \frac{1}{2}\left(-\frac{1}{2\left(p_{i+1}-p_i\right)} - \frac{1}{2\left(p_i - p_{i-1}\right)}\right)$$

If r_i is so low that an adjacent party gets no vote at all, say party $i+1$, then the vote share of party i becomes

$$\alpha_i = \frac{1}{2}\left(\frac{r_{i+2}-r_i}{2\left(p_{i+2}-p_i\right)} - \frac{r_i - r_{i-1}}{2\left(p_i - p_{i-1}\right)}\right)$$

Also in this case, α_i is linear in r_i, and

$$\frac{\partial \alpha_i}{\partial r_i} = \frac{1}{2}\left(-\frac{1}{2\left(p_{i+2}-p_i\right)} - \frac{1}{2\left(p_i - p_{i-1}\right)}\right)$$

Hence, the vote share function has a kink at r_i just low enough so that an adjacent party gets no vote at all. However, note that the vote share function has now lower slope in absolute value since $p_{i+2} > p_{i+1}$. The same logic applies when r_i is so low that even party $i+2$ gets no vote at all, and so on.

When r_i is low enough so that even party m does not have any vote, then the final policy becomes $p = \frac{1}{2+p_i-p_1}\left(p_1 + p_i + \frac{r_1 - r_i}{2}\right)$ and the vote share of party i becomes

$$\alpha_i = \frac{1}{2}\left(1 - \left(\frac{1}{2+p_i-p_1}\left(p_1 + p_i + \frac{r_1 - r_i}{2}\right) + \frac{r_i - r_{i-1}}{2\left(p_i - p_{i-1}\right)}\right)\right)$$

Also in this case, α_i is linear in r_i, and

$$\frac{\partial \alpha_i}{\partial r_i} = \frac{1}{2}\left(\frac{1}{2\left(2+p_i-p_1\right)} - \frac{1}{2\left(p_i - p_{i-1}\right)}\right)$$

Hence, the vote share function has a kink at r_i just low enough so that even party m gets no vote at all. However, note that the vote share function has now lower slope in absolute value since $\frac{1}{2(2+p_i-p_1)} > 0$.

To sum up, the demand function between the kinks is linear and the slope of a segment between two kinks is higher than the slope of the next segment. The same argument applies when some parties to the left of party i do not have any vote or when some parties both to the left and to the right of party i do not have any vote. Hence, α_i is concave in r_i for $i = 2, 3, ..., m-1$.

For party 1,

$$\alpha_1 = \frac{1}{2}\left(e_1 - (-1)\right) = \frac{1}{2}\left(\frac{1}{2+p_m-p_1}\left(p_1+p_m+\frac{r_1-r_m}{2}\right)+\frac{r_2-r_1}{2\left(p_2-p_1\right)}+1\right)$$

given that party 2 gets some votes. In this case, α_1 is linear in r_1, and

$$\frac{\partial \alpha_1}{\partial r_1} = \frac{1}{2}\left(\frac{1}{2\left(2+p_m-p_1\right)} - \frac{1}{2\left(p_2-p_1\right)}\right)$$

If r_1 is so low that party 2 gets no vote at all, then the vote share of party 1 becomes

$$\alpha_1 = \frac{1}{2}\left(\frac{1}{2+p_m-p_1}\left(p_1+p_m+\frac{r_1-r_m}{2}\right)+\frac{r_3-r_1}{2\left(p_3-p_1\right)}+1\right)$$

Also in this case, α_1 is linear in r_1, and

$$\frac{\partial \alpha_1}{\partial r_1} = \frac{1}{2}\left(\frac{1}{2\left(2+p_m-p_1\right)} - \frac{1}{2\left(p_3-p_1\right)}\right)$$

Hence, the vote share function has a kink at r_1 just low enough that party 2 gets no vote at all. However, note that the vote share function has now lower slope in absolute value since $p_3 > p_2$. The same line of reasoning applies when r_1 is so low that even party 3 gets no vote at all, and so on.

To sum up, the demand function between the kinks is linear and the slope of a segment between two kinks is higher than the slope of the next segment. Hence, α_1 is concave in r_1.

Similarly, it can be shown that α_m is concave in r_m. Hence, α_i is concave in r_i for every i. Existence of an equilibrium follows from Rosen (1965).

Now, we show that the equilibrium is unique. Using equation (5), we can express r_3 as a linear function of r_1 and r_2, calling this function as R^3, $r_3 = R^3(r_1, r_2)$. Similarly, $r_4 = R^4(r_2, r_3)$. Replacing r_3, this becomes $r_4 = R^4(r_2, R^3(r_1, r_2))$ which can be written as $r_4 = h^4(r_1, r_2)$. Given that R^3 and R^4 are linear, h^4 is linear too. Continuing this procedure, we have $r_{m-1} = h^{m-1}(r_1, r_2)$ and $r_m = h^m(r_1, r_2)$, both h^{m-1} and h^m being linear. Replacing these values on equations (7) and (8), we obtain two linear equations of two unknowns r_1 and r_2. We have already proved that an equilibrium exists. Hence, if there is not a unique equilibrium, then the two linear equations on r_1 and r_2 should coincide and every point of this straight line should be an equilibrium. Specifically, there should be an equilibrium where

$r_1 = 0$ or $r_2 = 0$, as any given line must intersect one of the axes[8]. However, this contradicts with the result of the last section proving that in any equilibrium, all parties choose positive rents. Then, there exists a unique pair of equilibrum rents r_1^* and r_2^*. Since, r_i, $i = 3, ..., m$, is uniquely defined in terms of r_1 and r_2 by the procedure explained above, there is a unique set of equilibrium rents $(r_1^*, r_2^*, ..., r_m^*)$. **Q.E.D.**

Proof of Proposition 5: *(i)* Assume m is odd and the parties' ideal policy points are symmetric around $p_{\frac{m+1}{2}} = 0$. Let's define δ_0 as $\delta_0 = p_{\frac{m+1}{2}} - p_{\frac{m-1}{2}} = p_{\frac{m+3}{2}} - p_{\frac{m+1}{2}}$, δ_1 as $\delta_1 = p_{\frac{m+5}{2}} - p_{\frac{m+3}{2}} = p_{\frac{m-1}{2}} - p_{\frac{m-3}{2}}$, and so on. Then, from equation (5),

$$r_{\frac{m+1}{2}} = \frac{1}{2}\left(\frac{\delta_0 r_{\frac{m+3}{2}} + \delta_0 r_{\frac{m-1}{2}}}{2\delta_0}\right)$$

Given the symmetric structure of the game and the uniqueness of equilibrium, the equilibrium will be such that $r_1 = r_m$, $r_2 = r_{m-1}$, etc. (Formally, $r_i = r_{m-i+1}$ for every i)

Hence, $r_{\frac{m+3}{2}} = r_{\frac{m-1}{2}}$. Then the above equation becomes simply

$$r_{\frac{m+3}{2}} = r_{\frac{m-1}{2}} = 2r_{\frac{m+1}{2}} \tag{10}$$

Assume m is even and the parties' ideal policy points are symmetric around 0. Let's define δ_0 as $\delta_0 = p_{\frac{m}{2}+1} - p_{\frac{m}{2}}$ and δ_1 as $\delta_1 = p_{\frac{m}{2}+2} - p_{\frac{m}{2}+1} = p_{\frac{m}{2}} - p_{\frac{m}{2}-1}$, and so on. Then, from equation (5),

$$r_{\frac{m}{2}+1} = \frac{1}{2}\left(\frac{\delta_0 r_{\frac{m}{2}+2} + \delta_1 r_{\frac{m}{2}}}{\delta_0 + \delta_1}\right)$$

Given the symmetric structure of the game and the uniqueness of equilibrium, the equilibrium will be such that $r_1 = r_m$, $r_2 = r_{m-1}$, etc. (Formally, $r_i = r_{m-i+1}$ for every $i \in I$)

Hence, $r_{\frac{m}{2}} = r_{\frac{m}{2}+1}$. Then the above equation becomes simply

$$r_{\frac{m}{2}+2} = \frac{2\delta_0 + \delta_1}{\delta_0} r_{\frac{m}{2}+1}$$

Hence, we conclude that

$$r_{\frac{m}{2}+2} > 2r_{\frac{m}{2}+1} \tag{11}$$

Now, we suppose that $r_i \geq 2r_{i-1}$ and we will show that $r_{i+1} \geq 2r_i$. Then, the first result of the proposition follows from equation (10) for m odd, and from equation (11) for m even.

[8] An equilibrium where $r_1 = 0$ and $r_2 = 0$ is impossible, since there are constants in equations (7) and (8).

We write equation (5) for i. This gives

$$r_i = \frac{1}{2}\left(\frac{\delta_{i-1}r_{i+1} + \delta_i r_{i-1}}{\delta_{i-1} + \delta_i}\right)$$

This is equivalent to

$$\delta_{i-1}r_{i+1} = 2\delta_{i-1}r_i + 2\delta_i r_i - \delta_i r_{i-1}$$

Since $r_i \geq 2r_{i-1}$, $2\delta_i r_i \geq 4\delta_i r_{i-1}$. Hence, $\delta_{i-1}r_{i+1} \geq 2\delta_{i-1}r_i + 3\delta_i r_{i-1}$. Then, we conclude that $r_{i+1} \geq 2r_i$.

Hence, we have shown that a party chooses a rent level at least twice as high as its more moderate adjacent party.

(ii) Now, we will show that a party's vote share is higher than its more moderate adjacent party for $m \geq 4$.

From equations (5) and (6), we find the following relation between party i's optimal rent level and vote share:

$$\alpha_i = \frac{p_{i+1} - p_{i-1}}{4\left(p_{i+1} - p_i\right)\left(p_i - p_{i-1}\right)} r_i \tag{12}$$

Combining equations (5) and (12), we obtain the following relation between the vote shares of three adjacent parties:

$$\alpha_i = \frac{1}{2}\left(\frac{p_{i+2} - p_{i+1}}{p_{i+2} - p_i}\alpha_{i+1} + \frac{p_{i-1} - p_{i-2}}{p_i - p_{i-2}}\alpha_{i-1}\right) \tag{13}$$

Assume m is odd. Writing equation (13) for $i = \frac{m+1}{2}$ for $m \geq 5$ [9],

$$\alpha_{\frac{m+1}{2}} = \frac{1}{2}\left(\frac{p_{\frac{m+5}{2}} - p_{\frac{m+3}{2}}}{p_{\frac{m+5}{2}} - p_{\frac{m+1}{2}}}\alpha_{\frac{m+3}{2}} + \frac{p_{\frac{m-1}{2}} - p_{\frac{m-3}{2}}}{p_{\frac{m+1}{2}} - p_{\frac{m-3}{2}}}\alpha_{\frac{m-1}{2}}\right)$$

It is equivalent to

$$\alpha_{\frac{m+1}{2}} = \frac{1}{2}\left(\frac{\delta_1}{\delta_0 + \delta_1}\alpha_{\frac{m+3}{2}} + \frac{\delta_1}{\delta_0 + \delta_1}\alpha_{\frac{m-1}{2}}\right)$$

By symmetry, $\alpha_{\frac{m+3}{2}} = \alpha_{\frac{m-1}{2}}$. Hence,

$$\alpha_{\frac{m+1}{2}} = \frac{\delta_1}{\delta_0 + \delta_1}\alpha_{\frac{m+3}{2}}$$

[9] The equation below is not defined for $m < 5$.

Hence,

$$\alpha_{\frac{m+3}{2}} > \alpha_{\frac{m+1}{2}} \tag{14}$$

Assume m is even. Writing equation (13) for $i = \frac{m}{2} + 1$ for $m \geq 6$ [10],

$$\alpha_{\frac{m}{2}+1} = \frac{1}{2}\left(\frac{\delta_2}{\delta_1 + \delta_2}\alpha_{\frac{m}{2}+2} + \frac{\delta_1}{\delta_0 + \delta_1}\alpha_{\frac{m}{2}}\right)$$

This gives

$$\frac{\delta_2}{\delta_1 + \delta_2}\alpha_{\frac{m}{2}+2} = 2\alpha_{\frac{m}{2}+1} - \frac{\delta_1}{\delta_0 + \delta_1}\alpha_{\frac{m}{2}} > \alpha_{\frac{m}{2}+1}$$

By symmetry, $\alpha_{\frac{m}{2}+1} = \alpha_{\frac{m}{2}}$. Then, the last inequality follows from $\alpha_{\frac{m}{2}+1} = \alpha_{\frac{m}{2}}$ and $\frac{\delta_1}{\delta_0+\delta_1} < 1$.

Since $\frac{\delta_2}{\delta_1+\delta_2} < 1$, it follows from the inequality that

$$\alpha_{\frac{m}{2}+2} > \alpha_{\frac{m}{2}+1} \tag{15}$$

Now, we suppose that $\alpha_i > \alpha_{i-1}$ and we will show that $\alpha_{i+1} > \alpha_i$.

Writing equation (13) for i such that $3 \leq i \leq m - 2$ [11],

$$\alpha_i = \frac{1}{2}\left(\frac{\delta_{i+1}}{\delta_{i+1} + \delta_i}\alpha_{i+1} + \frac{\delta_{i-2}}{\delta_{i-2} + \delta_{i-1}}\alpha_{i-1}\right)$$

Using $\alpha_i > \alpha_{i-1}$, this gives

$$\left(2 - \frac{\delta_{i-2}}{\delta_{i-2} + \delta_{i-1}}\right)\alpha_i < \frac{\delta_{i+1}}{\delta_{i+1} + \delta_i}\alpha_{i+1}$$

Since the coefficient of α_i is at least as high as the coefficient of α_{i+1}, we have $\alpha_{i+1} > \alpha_i$.

Note that since we could write equation (13) only for $3 \leq i \leq m-2$, we still have to show that $\alpha_m > \alpha_{m-1}$. By symmetry, $\alpha_1 > \alpha_2$ will follow.

From above, we know that

$$r_m \geq 2r_{m-1} + \frac{3\delta_{m-1}}{\delta_{m-2}}r_{m-2}$$

[10] The equation below is not defined for $m < 6$.

[11] Otherwise, the equation below is not defined.

Writing equation (5) for $m-1$,

$$r_{m-1} = \frac{\delta_{m-1} r_{m-2} + \delta_{m-2} r_m}{2\left(\delta_{m-1} + \delta_{m-2}\right)}$$

Manipulating together the last two equations, we get

$$r_m \geq \left(\frac{1}{2} + \frac{3\left(\delta_{m-1} + \delta_{m-2}\right)}{2\delta_{m-2}}\right) r_{m-1}$$

Similarly as we did for $i \neq 1, m$, we find the following relation between party m's optimal rent level and vote share:

$$\alpha_m = \frac{2 + p_{m-1} - p_1}{4\left(2 + p_m - p_1\right)\left(p_m - p_{m-1}\right)} r_m \tag{16}$$

Combining the last two equations, we get

$$\alpha_m \geq \left(\frac{4 - \delta_{m-1}}{16\delta_{m-1}}\right)\left(\frac{1}{2} + \frac{3\left(\delta_{m-1} + \delta_{m-2}\right)}{2\delta_{m-2}}\right) r_{m-1}$$

Writing equation (12) for $m-1$,

$$\alpha_{m-1} = \frac{\delta_{m-1} + \delta_{m-2}}{4\delta_{m-1}\delta_{m-2}} r_{m-1}$$

Comparing the last two equations, it can be shown that $\alpha_m > \alpha_{m-1}$.

Hence, we have proven that a party's vote share is higher than its more moderate adjacent party for $m \geq 5$.

For $m = 4$, using equations (5), (12) and (16), it can be similarly shown that $\alpha_1 = \alpha_4 > \alpha_2 = \alpha_3$. Hence, the above result is also true for $m = 4$.

(iii) Since a party's payoff is $\alpha_i r_i$, the third result follows from the first two for $m \geq 4$. **Q.E.D.**

Proof of Proposition 6: We know from the proof of Proposition 2 that, for $i \neq 1, m$,

$$r_i < \frac{\left(p_i - p_{i-1}\right) r_{i+1} + \left(p_{i+1} - p_i\right) r_{i-1}}{p_{i+1} - p_{i-1}}$$

Using this inequality and the same steps as in the proof of part (i) of Proposition 5, the result follows. **Q.E.D.**

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