

# Variational regularization under conditional stability estimates and the distinguished case of oversmoothing penalties

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## Abstract:

Conditional stability estimates require additional regularization for obtaining stable approximate solutions if the validity area of such estimates is not completely known. The focus of this talk is on the Tikhonov regularization under conditional stability estimates for non-linear ill-posed problems in Hilbert scales, where the case that the penalty is oversmoothing plays a prominent role. This oversmoothing problem has been early studied for linear forward operators, most notably in the seminal paper by Natterer [1]. The a priori parameter choice used there, just providing order optimal convergence rates, has in the oversmoothing case the unexpected property that the quotient of the noise level square and the regularization parameter tends to infinity when the noise level tends to zero. Taking into account the recent paper [2], where results on oversmoothing penalties have been extended to non-linear forward operators, we provide in this talk some new case studies along the lines of [3] that enlighten the interplay of conditional stability and regularization. In particular, there occur pitfalls for oversmoothing penalties, because convergence can completely fail and the stabilizing effect of conditional stability may be lost. Numerical case studies for some nonlinear examples illustrate such effects.

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## References

- [1] F. NATTERER: Error bounds for Tikhonov regularization in Hilbert scales. *Applicable Analysis*, 18:29–37, 1984.
- [2] B. HOFMANN, P. MATHÉ: Tikhonov regularization with oversmoothing penalty for non-linear ill-posed problems in Hilbert scales. *Inverse Problems*, 34:015007 (14pp), 2018.
- [3] D. GERTH, B.HOFMANN, C.HOFMANN: Case studies and a pitfall for nonlinear variational regularization under conditional stability. Paper in preparation, 2019.